

**Holomorphic semigroups and Sarason's
characterization of vanishing mean oscillation**
(this is a joint work with N. Chalmoukis)

Abstract

It is a classical theorem of Sarason that an analytic function of bounded mean oscillation ($BMOA$), is of vanishing mean oscillation ($VMOA$) if and only if its rotations converge in norm to the original function as the angle of the rotation tends to zero, i.e. for a function $f \in BMOA$,

$$\lim_{t \rightarrow 0^+} \|f \circ e^{it}z - f\|_{BMOA} = 0$$

if and only if $f \in VMOA$.

In a series of two papers [1, 2] Blasco et al. have raised the problem of characterizing all semigroups of holomorphic functions (φ_t) that can replace the semigroup of rotations in Sarason's Theorem. We give a complete answer to this question, in terms of a "logarithmic vanishing oscillation" condition on the infinitesimal generator of the semigroup (φ_t) . In addition we confirm the conjecture of Blasco et al. [2, Question 2] that all such semigroups are elliptic. We also investigate the analogous question for the Bloch and the little Bloch space and, surprisingly enough, we find that the semigroups for which the Bloch version of Sarason's Theorem holds are exactly the same as in the $BMOA$ case.

REFERENCES

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