9th Greek Algebra and Number Theory Conference

Department of Mathematics, Aristotle University of Thessaloniki

12-13 May 2023

Program of Talks

Friday, 12 May: Teloglion Foundation	
9:00 - 9.30	**Welcome**
9:30 - 10:00	**Dimitrios Poulakis:** *Integral points of conics over number fields*
10:00 - 10:30	**Evis Ieronymou:** *Brauer-Manin obstruction: Overview and some recent results on K3 surfaces*
10:30 - 11:00	**Kostas Karagiannis:** *Representations on canonical models of generalized Fermat curves and their syzygies*
11:00 - 11:30	**Coffee break**
11:30 - 12:00	**Konstantinos Kofinas:** *On automorphisms of certain free nilpotent-by-abelian Lie algebras*
12:00 - 12:30	**Panagis Karazeris:** *Point-free topology and internal locales in a topos*
12:30 - 13:00	**Bob Coecke:** *From categorical quantum mechanics to quantum tech... and quantum for kids*
13:00 - 15:00	**Lunch Break**
15:00 - 15:30	**Ioannis Emmanouil:** *Orthogonality in homotopy categories*
15:30 - 16:00	**Dionysia Stergiopoulou:** *Projectively coresolved Gorenstein flat dimension of groups*
16:00 - 16:30	**Mihalis Maliakas:** *Homomorphisms and extensions between Weyl modules*
16:30 - 17:00	**Coffee Break**
17:00 - 17:30	**Maria Loukaki:** *On the common transversal probability in finite groups (Part A)*
17:30 - 18:00	**Stefanos Aivazidis:** *On the common transversal probability in finite groups (Part B)*
18:00 - 18:30	**Rizos Sklinos:** *First-order sentences in random groups*
18:30 - 19:00	**Discussion about the next conference**
21:00 - ∞	**Conference dinner**

Saturday, 13 May: Room M2, 3rd floor, School of Sciences	
10:00 - 10:30	**Maria Chlouveraki:** *Generalizing the Temperley-Lieb algebra*
10:30 - 11:00	**Vasiliki Petrotou:** *The 4-intersection format with an application to Fano 3-folds*
11:00 - 11:30	**Sofia Lambropoulou:** *From plat closure to standard closure of braids*
11:30 - 12:00	**Coffee break**
12:00 - 12:30	**Dimitra Kosta:** *On the strongly robust property of toric ideals*
12:30 - 13:00	**Christos Tatakis:** *The structure of complete intersection graphs and their planarity*
13:00 - 13:30	**Apostolos Thoma:** *Theory and applications of bouquet toric ideals*
13:30 - 15:30	**Lunch Break**
15:30 - 16:00	**Mihalis Kolountzakis:** *Decidability questions for tiling and spectral set problems*
16:00 - 16:30	**Dimitrios Chatzakos:** *The QUE problem for degenerate Eisenstein series of higher rank*
16:30 - 17:00	**Giorgios Kapetanakis:** *The line and the translate properties for r-primitive elements*
17:00 - 17:30	**Coffee Break**
17:30 - 18:00	**Eleni Tzanaki:** *On the enumeration of Shi regions in Weyl cones*
18:00 - 18:30	**Stavros Papadakis:** *The Hibi-Ohsugi conjecture for IDP Gorenstein Lattice Polytopes*

Abstracts

Stefanos Aivazidis
Univeristy of Crete

On the common transversal probability in finite groups (Part B)

We build on Maria Loukaki's previous talk to define and analyse a global invariant, depending only the parent group G, which we call the transversal probability of G and write as $\mathrm{tp}(G)$. We will see that $\mathrm{tp}(G)$ satisfies various nice abstract properties like subgroup- and quotient-monotonicity and thus serves as an efficient "detector" of key properties of G. Roughly, this means that the larger $\mathrm{tp}(G)$ is, the more normal structure G exhibits.

Joint work with Maria Loukaki and Thomas Müller.

Dimitrios Chatzakos
University of Patras

The QUE problem for degenerate Eisenstein series of higher rank

The QUE conjecture for the classical Eisenstein series attached to the modular group was proved by Luo and Sarnak, whereas the refined quantum variance problem was studied recently by Huang. The QUE conjecture for degenerate Eisenstein series on $\mathrm{GL}(n)$ was proved by Zhang. In this walk we will briefly discuss the quantum variance problem for the degenerate Eisenstein series. This is a joint work in progress with Corentin Darreye.

Maria Chlouveraki
National and Kapodistrian University of Athens

Generalizing the Temperley-Lieb algebra

The Temperley-Lieb algebra was introduced by Temperley and Lieb for its applications in statistical mechanics. It has several definitions, one of which is via a quotient of the Iwahori-Hecke algebra of type A. It is thanks to this definition that Jones was able to use the Temperley-Lieb algebra to define the famous knot invariant known as the Jones polynomial. Attempts to generalize the notion of the Temperley-Lieb algebra to other types have been numerous, some more successful than others. In the past 10 years, the Yokonuma-Hecke algebra of type A, which is a generalization of the Iwahori-Hecke algebra of the same type, came into the spotlight for its topological applications. In this talk, we will study 3 possible candidates for the generalization of the Temperley-Lieb algebra in this context and declare a winner. This is joint work with Guillaume Pouchin.

Bob Coecke
University of Oxford

From categorical quantum mechanics to quantum tech...and quantum for kids

Categorical quantum mechanics started as a proposed answer to von Neumann's desire for another quantum formalism. In fairly short time it became a full-blown alternative to Hilbert space: on the one hand, it worked for all practical purposes [1], and on the other hand, thanks to a 'logical' completeness theorem [2], that is, any equation derivable in Hilbert space maths is derivable in CQM. The particular appeal of CQM comes from its purely graphical calculus, which is now also the manner in which it is presented [1]. It has now become widespread in quantum industry, and at the same time it has become an educational tool at high-school level [3]. A remarkable correspondence between CQM and linguistic structure has resulted in practical quantum natural language processing [4].

[1] Bob Coecke and Aleks Kissinger (2017) Picturing Quantum Processes. Cambridge University Press.
[2] https://dl.acm.org/doi/10.1145/3209108.3209128
[3] Bob Coecke and Stefano Gogioso (2022) Quantum in Pictures. Quantinuum.
[4] https://www.forbes.com/sites/moorinsights/2021/10/13/cambridge-quantum-makes-quantum-natural-language-processing-a-reality/

Ioannis Emmanouil
National and Kapodistrian University of Athens

Orthogonality in homotopy categories

We plan to describe three examples of orthogonal pairs in the homotopy category of a ring and explain their role in the framework of Gorenstein homological algebra.

Evis Ieronymou
University of Cyprus

Brauer-Manin obstruction: Overview and some recent results on $K3$ surfaces

A fundamental question in arithmetic geometry is to understand the set of rational points of algebraic varieties over number fields. The theory of the Brauer-Manin obstruction is concerned with qualitative questions regarding the aforementioned set. In this talk we give an overview of this theory, and report on some recent progress for some classes of $K3$ surfaces.

Giorgios Kapetanakis
University of Thessaly

The line and the translate properties for r-primitive elements

Let q be a prime power and $n \geq 2$ an integer. We denote by $\mathbb{F}_q$ the finite field of q elements and by $\mathbb{F}_{q^n}$ its extension of degree n. An element of $\mathbb{F}_{q^n}^*$ of order $(q^n-1)/r$, where $r \mid q^n - 1$, is called *r-primitive*, while, if $r = 1$, we simply call it *primitive*.

If θ is a *generator* of the extension $\mathbb{F}_{q^n}/\mathbb{F}_q$, i.e., is such that $\mathbb{F}_{q^n} = \mathbb{F}_q(\theta)$, then

$$\mathcal{T}_\theta := \{\theta + x \, : \, x \in \mathbb{F}_q\}$$

is the *set of translates* of θ over $\mathbb{F}_q$ and, if $\alpha \in \mathbb{F}_{q^n}^*$,

$$\mathcal{L}_{\alpha,\theta} := \{\alpha(\theta + x) \, : \, x \in \mathbb{F}_q\}$$

is the *line* of α and θ over $\mathbb{F}_q$. It is known that, given n, if q is large enough, every set of translates and every line contain a primitive element, while effective versions for these existence results are known for just a few small values of n.

In this work, we extend these existence results to r-primitive elements and we provide effective results for the case $r = n = 2$.

This is joint work with Stephen D. Cohen.

Kostas Karagiannis
University of Manchester

Representations on canonical models of generalized Fermat curves and their syzygies

The homogeneous coordinate ring S_X of a smooth curve X relative to a projective embedding carries rich algebraic properties which reflect much of the geometry of X. These properties can be split into three categories, each corresponding to a different algebraic structure on S_X: that of a graded algebra over the ground field, a representation for the curve's automorphism group, or a module over a polynomial ring - the coordinate ring of the ambient space. In this talk, I will present recent work on an explicit construction which unifies all three structures under a common theme, focusing on a family of curves which generalizes Fermat's equation $x^n + y^n + z^n = 0$.

Panagis Karazeris
University of Patras

Point-free topology and internal locales in a topos

Locales constitute the correct substitute for topological spaces in mathematical universes, such as toposes, where points may occur scarcely, as their existence may depend on non-constructive principles like the axiom of choice or the weaker prime ideal theorem for distributive lattices, which fail in such universes. They appear as duals of structures such as commutative rings, distributive lattices, or C^*-algebras even when these structures lack sufficiently many prime or maximal ideals. They also allow us to talk about fundamental structures in such universes, like the real numbers, as spaces which maintain the right properties, where their construction as Dedekind cuts or Cauchy sequence may produce non-isomorphic results in the absence of the axiom of choice.

They occur as collections of open sets, forming a complete distributive lattice (frame). The points come as a secondary notion and their classically conceived role of determining the space is a consequence of foundational principles, logical or set-theoretic, that we adopt. Moreover these complete distributive lattices of open subsets occur much more naturally and become more transparent than when trying to impose a topology on some set of points. For example, the frame of open subsets for the Zariski spectrum of a commutative ring is that of radical ideals while the Balmer spectrum of a tensor triangulated category is described directly in terms of the frame of radical thick tensor ideals using the Hochster dual of a coherent frame, which becomes an entirely elementary idea in this setting.

The absence of point-set arguments in this theory is compensated by the availability of methods of (essentially, commutative) algebra: Frames are algebraic structures and we can speak of free frames, congruences (which are encoded by certain endomaps of the frame known as nuclei), presentations and constructions of coproducts of frames (= products in the category of locales) via tensor products in the broader monoidal category of complete lattices with maps that preserve suprema. These "commutative algebraic" methods allow for the determination of the category $\mathrm{Loc}(\mathrm{Shv}\,X)$ of locales inside a topos of sheaves over a locale X as Loc/X, the category of locales fiberwise over X and subsequently translating results that hold in the internal logic of a topos into results about continuous maps of topological spaces. We exemplify this by showing the analogue of the well-known topological result, that a closed quotient of a compact Hausdorff locale is compact Hausdorff and then suitably translating it. On the other hand, results about certain properties of localic maps (e.g, the stability of open maps under pullback) make possible the description of constructs of internal locales (for example, that products of locales in a presheaf topos are given object-wise) and subsequently the transfer of properties (e.g, being Hausdorff) from internal locales to their sections and vice-versa.

Konstantinos Kofinas
University of the Aegean

On automorphisms of certain free nilpotent-by-abelian Lie algebras

For a positive integer $n \geq 4$, let R_n be a free (nilpotent of class 2)-by-abelian and abelian-by-(nilpotent of class 2) Lie algebra of rank n. We show that the subgroup of $\mathrm{Aut}(R_n)$ generated by the tame automorphisms and a countably infinite set of explicitly given automorphisms of R_n is dense in $\mathrm{Aut}(R_n)$ with respect to the formal power series topology.

Mihalis Kolountzakis
University of Crete

Decidability questions for tiling and spectral set problems

A finite set $A \subseteq \mathbb{Z}^d$ is a *translational* tile if we can translate it at locations $B \subseteq \mathbb{Z}^d$ such that every point in $\mathbb{Z}^d$ is covered exactly once by the translates of A. In other words, each $x \in \mathbb{Z}^d$ is written uniquely as $x = a + b$, $a \in A$, $b \in B$.

We are interested to algorithmically decide the question "Is A a tile?". Because of the infinity of the space to be tiled this is not a trivial question, even for $d = 1$. At this point we do not care about complexity issues, just computability, however slow.

It turns out that this question is intimately linked to the question of periodicity of the *tiling complement* B and the so-called "Periodic tiling conjecture", which states that if A is a tile then it must also have a periodic tiling complement B (not every tiling complement needs to be periodic though). A set B is called periodic if $B = B + g$, for $g \in G$, where $G \subseteq \mathbb{Z}^d$ is a subgroup of finite index (a lattice).

It has long been known (Berger, Wang, in the 1960s) that the Periodic tiling conjecture implies decidability of the tiling. This conjecture was proved for $d = 2$ in 2016 by S. Bhattacharya and more recently also by R. Greenfeld and T. Tao. Last year, however, R. Greenfeld and T. Tao also managed to disprove the conjecture in high enough dimension. So, although we do have a decision procedure in dimensions $d = 1, 2$, we do not know of such a method in higher dimension.

The related question "Can A tile periodically?" is identical to the original question in dimensions where the Periodic tiling conjecture holds, but is a different question in high enough dimension because of the disproof by R. Grenefeld and T. Tao. It is also unknown if this question can be answered algorithmically in dimension $d > 2$.

Translational tiling has been studied almost in parallel in the last 40 years with the so-called *spectral set* problem. A finite set $A \in \mathbb{Z}^d$ is called *spectral* if one can find an orthogonal basis for $\ell^2(A)$ (the complex functions on A with the usual inner product) consisting of characters of ^{d}, i.e. of functions $e_x(n) = n \to e^{2\pi i x \cdot n}$, where is $x \in \mathbb{T}^d = \mathbb{R}^d/\mathbb{Z}^d$. The connection between tiling and spectrality was the "Fuglede conjecture": *A is a tile if and only if A is spectral.*

This is known since 2003-4 (by T. Tao, M. Matolcsi and myself) to be false in high dimension (this dimension is now down to $d \geq 3$, for both directions of the conjecture). But, as dead conjectures go, this has proved a very resilient statement that has spawned a lot of research, even after death. For example, the conjecture, properly stated in the ambient space $\mathbb{R}^d$ instead of $\mathbb{Z}^d$, is known to be true in all dimensions if A is a convex set (M. Matolcsi and N. Lev). And a lot of work is still being done on establishing or refuting the unrestricted conjecture in dimensions 1 and 2 and also in different host groups (both tiling and spectrality make perfect sense in more general abelian groups).

The corresponding decidability question "Is A spectral?" shows many similarities to the tiling decidability question. In some sense the periodicity of the tiling is replaced here by the question of the *rationality* if the spectrum, which is still an open question even in dimension 1.

In this talk I hope to present some background material and show many of the connections between tiling and spectrality especially from the point of view of decidability. Some recent progress (mostly joint with R. Malikiosis) will also be described.

Dimitra Kosta
University of Edinburgh

On the strongly robust property of toric ideals

To every toric ideal one can associate a structure, consisting of a graph and another toric ideal, called bouquet ideal. The connected components of this graph are called bouquets. Bouquets are of three types; free, mixed and non-mixed. We prove that the cardinality of the following sets - the set of indispensable elements, minimal Markov bases, the Universal Markov basis and the Universal Gröbner basis of a toric ideal - depends only on the type of the bouquets and the bouquet ideal. These results enable us to introduce the strongly robust simplicial complex and show that it determines the strongly robust property. For codimension 2 toric ideals, we study the strongly robust simplicial complex and prove that robust implies strongly robust. This is joint work with A. Thoma and M. Vladoiu.

Sofia Lambropoulou
National Technical University of Athens

From plat closure to standard closure of braids

An element of the Artin braid group may represent a knot or link in $\mathbb{R}^3$ by employing the standard closure or the plat closure and every knot or link can be represented in either way. This is also the case for knots and links in a handlebody or in a thickened surface. Each type of closure has its advantages. For example the plat closure is most suitable when constructing quantum invariants or when representing a c.c.o. 3-manifold by its handle decomposition, whilst the standard closure is most suitable when constructing Jones-type invariants or when representing a c.c.o. 3-manifold by a surgery description. It is, thus, important to be able to pass from one type of closure to the other. In this work we describe an algorithm for passing from a braid in plat form to a braid in the standard closure representing the same knot or link, and vice-versa. We analyze the three cases of links: in $\mathbb{R}^3$, in handlebodies and in thickened surfaces. We show that the algorithm is quadratic in the number of crossing generators and loop generators of the classical, mixed or surface braid group respectively, when passing from plat to standard closure, while it is linear when passing from standard to plat closure.

This is joint work with Paolo Cavicchioli (U Modena)

Maria Loukaki
University of Crete

On the common transversal probability in finite groups (Part A)

Let G be a finite group and H a subgroup of G. What is the probability $P_G(H)$ of a right transversal of H in G to be a left transversal as well? We will give a method to compute precisely $P_G(H)$ with the help of, the so called, coset intersection graph. A limit theorem for $P_G(H)$ is given and a new invariant $tp(G)$ for the group G is introduced. This is joint work with S. Aivazidis and T. Muller.

Mihalis Maliakas
National and Kapodistrian University of Athens

Homomorphisms and extensions between Weyl modules

For an infinite field K of positive characteristic p and a positive integer r, let $S = S_K(n, r)$ denote the corresponding Schur algebra of $G = GL_n(K)$. The category of finite dimensional $S_K(n, r)$-modules is equivalent to the category of homogeneous polynomial representations of G of degree r. Several important $S_K(n, r)$-modules are indexed by partitions λ of r with at most n parts, such as the Weyl modules $\Delta(\lambda)$ and the simple modules $L(\lambda)$. The study of extension groups between such modules is one of the main problems in the polynomial representation theory of G. There are relatively few general results, especially those that relate extension groups corresponding to different degrees r. Motivated by a question of D. Hemmer, we examine in this talk periodicity phenomena related to the dimensions of $Ext^i_S(\Delta(\lambda), \Delta(\mu))$ and $Ext^i_S(\Delta(\lambda), L(\mu))$ when cells are added to the first parts of λ and μ. Also we examine stability of the dimension of $Hom_S(\Delta(\lambda), \Delta(\mu))$ when a partition γ is added to λ and μ such that sufficiently large powers of p divide the parts of γ, and we obtain a related non vanishing result. Consequences for the representation theory of the symmetric group will be presented. This talk is based on joint works with D.-D. Stergiopoulou and with Ch. Evangelou and D.-D. Stergiopoulou.

Stavros Papadakis
University of Ioannina

The Hibi-Ohsugi conjecture for IDP Gorenstein Lattice Polytopes

A lattice polytope P is a convex polytope whose vertices all have integer coordinates. Given a field k there is an associated commutative graded k-algebra $k[P]$. The polytope is called Gorenstein if $k[P]$ is Gorenstein and IDP if for all positive integers k every point with integer coordinates of the dilation polytope kP is a sum of k points of P with integer coordinates. Counting the number of points with integer coordinates of kP for all positive integers k leads to the notion of the h^*-vector of P. In 2006 Hibi and Ohsugi conjectured that the h^*-vector of an IDP Gorenstein lattice polytope is unimodal, which means that it never strictly increases after the first time it strictly decreases. The aim of the presentation is to discuss a recent proof of the conjecture which is a joint work with Karim Adiprasito, Vasiliki Petrotou and Johanna Steinmeyer.

Vasiliki Petrotou
Hebrew University of Jerusalem

The 4-intersection format with an application to Fano 3-folds

Unprojection is a theory due to Reid which constructs more complicated rings starting from simpler data. The idea of unprojection is intended for serial use. Papadakis and Neves developed a theory of parallel unprojection. In the present talk we develop a new method of unprojection. Starting from a codimension 2 complete intersection ideal, we use parallel unprojection of Kustin-Miller type in order to construct Gorenstein rings of codimension 6. We give three applications that construct families of Fano 3-folds embedded anticanonically in codimension 6.

Dimitrios Poulakis
Aristotle University of Thessaloniki

Integral points of conics over number fields

In this talk we will describe the computation of upper bounds of integral points on conics over number fields. In particular, we will examine the case of Pell equations.

1. Alvanos, Paraskevas; Poulakis, Dimitrios, *Bounds for the smallest integral point on a conic over a number field*, Acta Arith. 193, No. 4, 355-368 (2020).
2. Alvanos, Paraskevas; Poulakis, Dimitrios, *Bounds for the smallest integral solution of Pell equation over a number field*, Funct. Approximatio, Comment. Math. (to appear).

Rizos Sklinos
Chinese Academy of Sciences

First-order sentences in random groups

Gromov in his seminal paper introducing hyperbolic groups claimed that a "typical" finitely presented group is hyperbolic. His statement can be made rigorous in various natural ways. The model of randomness that is preferentially focused on is Gromov's density model, as it allows a fair amount of flexibility. In this model a random group is hyperbolic with overwhelming probability.

In a different line of thought, Tarski asked whether all non abelian free groups share the same first-order theory (in the language of groups). This question proved very hard to tackle and only after more than 50 years Sela and Kharlampovich-Myasnikov answered the question positively. Combining the two, J. Knight conjectured that a first-order sentence holds with overwhelming probability in a random group if and only if it is true in a no abelian free group. In joint work with O. Kharlampovich we answer the question positively for universal-existential sentences.

Dionysia Stergiopoulou
National and Kapodistrian University of Athens

Projectively coresolved Gorenstein flat dimension of groups

The Gorenstein projective and Gorenstein flat modules were introduced in 1995 by Enochs and Jenda and generalize the Auslander's modules of G-dimension zero.
Recently Saroch and Stovicek introduced a new class of modules, the projectively coresolved Gorenstein flat modules (or PGF-modules, for short). These modules are both Gorenstein projective and Gorenstein flat. Holm's metatheorem states that every result in classical homological algebra has a counterpart in Gorenstein homological algebra. However, the relation between Gorenstein projective and Gorenstein flat modules is not well understood. We believe that an analogue of projective modules in the Gorenstein homological algebra could be the PGF modules. The PGF dimension, which has been studied by Dalezios and Emmanouil, has many common properties with the Gorenstein projective dimension. In this talk we introduce and study the PGF dimension of the trivial RG-module R, where R is a commutative ring and G is a group. We show that this dimension enjoys all properties of the cohomological and the Gorenstein cohomological dimension of groups[a].

[a]Research supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "1st Call for H.F.R.I. Research Projects to support Faculty members and Researchers and the procurement of high-cost research equipment grant", project number 4226.

Christos Tatakis

Aristotle University of Thessaloniki

The structure of complete intersection graphs and their planarity (joint work with A.Thoma)

Let $A = \{\mathbf{a}_1, \ldots, \mathbf{a}_m\} \subseteq \mathbb{N}^n$ be a vector configuration in $\mathbb{Q}^n$ and

$$\mathbb{N}A := \{l_1\mathbf{a}_1 + \cdots + l_m\mathbf{a}_m \mid l_i \in \mathbb{N}\}$$

be the corresponding affine semigroup, where $\mathbb{N}A$ is pointed, that is if $x \in \mathbb{N}A$ and $-x \in \mathbb{N}A$ then $x = \mathbf{0}$. We grade the polynomial ring $\mathbb{K}[x_1, \ldots, x_m]$ over an arbitrary field $\mathbb{K}$ by the semigroup $\mathbb{N}A$ setting $\deg_A(x_i) = \mathbf{a}_i$ for $i = 1, \ldots, m$. For $\mathbf{u} = (u_1, \ldots, u_m) \in \mathbb{N}^m$, we define the A-degree of the monomial $\mathbf{x}^\mathbf{u} := x_1^{u_1} \cdots x_m^{u_m}$ to be

$$\deg_A(\mathbf{x}^\mathbf{u}) := u_1\mathbf{a}_1 + \cdots + u_m\mathbf{a}_m \in \mathbb{N}A.$$

The toric ideal I_A associated to A is the prime ideal generated by all the binomials $\mathbf{x}^\mathbf{u} - \mathbf{x}^\mathbf{v}$ such that $\deg_A(\mathbf{x}^\mathbf{u}) = \deg_A(\mathbf{x}^\mathbf{v})$.

Let G be a connected, undirected, finite, simple graph on the vertex set $V(G) = \{v_1, \ldots, v_n\}$ and let $E(G) = \{e_1, \ldots, e_m\}$ be the set of the edges of G. We denote by $\mathbb{K}[e_1, \ldots, e_m]$ the polynomial ring in the m variables $e_1, \ldots, e_m$ over an arbitrary field $\mathbb{K}$. We will associate each edge $e = (v_i, v_j) \in E(G)$ with the element $a_e = v_i + v_j$ in the free abelian group $\mathbb{Z}^n$, with basis the set of the vertices of G, where $v_i = (0, \ldots, 0, 1, 0, \ldots, 0)$ be the vector with 1 in the $i-$th coordinate of v_i. With I_G we denote the toric ideal I_{A_G} in $\mathbb{K}[e_1, \ldots, e_m]$, where $A_G = \{a_e \mid e \in E(G)\} \subset \mathbb{Z}^n$.

We study the complete intersection property on the toric ideal I_G.

In general, the toric ideal I_G is complete intersection if and only if it can be generated by h binomials, where $h = m - n + 1$ if G is a bipartite graph or $h = m - n$ if G is not a bipartite graph. The answer is known in the case of bipartite graphs, i.e. graphs with no odd cycles. In the last years, several useful partial results have been proved and they provide key properties of complete intersection toric ideals of graphs.

We focus on the general case and we present a structural theorem which gives us necessary and sufficient conditions in which the toric ideal I_G is complete intersection. Moreover, we characterize with sufficient and necessary conditions the complete intersection graphs which are planar.

Apostolos Thomas

University of Ioannina

Theory and applications of bouquet toric ideals

To any toric ideal I_A, encoded by an integer matrix A, we can associate an oriented matroid structure called the bouquet graph of A and another toric ideal called the bouquet ideal of A. The combination of these objects captures the essential combinatorial and algebraic information of the toric ideal I_A. Bouquets allow classification-type results and provide new ways to construct examples of toric ideals with various interesting properties. This talk is based on joint works with Dimitra Kosta, Shmuel Onn, Sonja Petrovic and Marius Vladoiu.

Eleni Tzanaki
University of Crete

On the enumeration of Shi regions in Weyl cones

Let Φ be an irreducible crystallographic root system with Weyl group W spanning a Euclidean space V. The reflection arrangement of W, which is the collection of hyperplanes $\langle a, x\rangle = 0$ with $a \in \Phi$, partitions the space V into cones known as *Weyl cones*. If Φ^+ is the positive part of Φ, the Shi arrangement is the collection of hyperplanes $\langle a, x\rangle = 0, 1$ with $a \in \Phi^+$, which partition V into *Shi regions*. Since the Shi arrangement contains the reflection arrangement, each Weyl cone is partitioned by Shi regions. Our main goal is to determine the number of Shi regions in each Weyl cone. To do so, we explain how Shi regions within each Weyl cone biject to antichains of a naturally-defined subposet of the root poset $(\Phi^+, \preceq)$. Then, we associate the root poset $(\Phi^+, \preceq)$ to an acyclic directed graph Γ_Φ so that antichains in each of the above subposets are in bijection with paths in Γ_Φ avoiding a certain collection of subpaths. We conclude with a determinental formula which resolves our path counting. This is joint work with Aram Dermenjian[a].

[a]The research project was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "2nd Call for H.F.R.I. Research Projects to support Faculty Members and Researchers"(Project Number:HFRI-FM20-04537)