The infinite Hilbert matrix on spaces of analytic functions

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The (finite) Hilbert matrix is arguably one of the single most well-known matrices in mathematics. The infinite Hilbert matrix $\mathcal{H}$ was introduced by David Hilbert around 120 years ago in connection with his double series theorem. It can be interpreted as a linear operator on spaces of analytic functions by its action on their Taylor coefficients. The boundedness of $\mathcal{H}$ on the Hardy spaces H^p for $1 < p < \infty$ and Bergman spaces A^p for $2 < p < \infty$ was established by Diamantopoulos and Siskakis. The exact value of the operator norm of $\mathcal{H}$ acting on the Bergman spaces A^p for $4 \leq p < \infty$ was shown to be $\frac{\pi}{\sin(2\pi/p)}$ by Dostanic, Jevtic and Vukotic in 2008 in combination with results of Diamantopoulos. The case $2 < p < 4$ was an open problem until in 2018 it was shown by Bozin and Karapetrovic that the norm has the same value also on the scale $2 < p < 4$. In this talk, we introduce some background and review some of the old results and consider the still partly open problem regarding the value of the norm on weighted Bergman spaces. We also consider a generalised Hilbert matrix operator and its (essential) norm. The talk is partly based on a joint work with Mikael Lindström, David Norrbo and Niklas Wikman (Åbo Akademi University).